**Solving equations and inequalities**

Taking our next steps in the vocabulary of the mathematics language, let us introduce some terms that are widely used:

**Equation:** Any mathematical statement that has an equal '=' sign in it. It has two interchangeable sides, referred to as the left and right hand sides. In some economic applications the norm is to assume that the right hand side causes the left. For example, if we find that Price=Marginal Costs, then from the point of view of a firm in a competitive market, they set the MC to equal the price and not the other way around. This direction of causation can be confusing and is hence best avoided and the sides kept interchangeable.

**Variable:** Any symbol, typically a letter of the alphabet, that is used to represent an unknown number. Eg. 5x+1=10. Here x is the variable. Some variables have conventional letters attached to them in economics, for example price and quantity are typically denoted by p and q respectively. But the choice of the symbol is completely arbitrary with the only condition being that different symbols be allocated to different unknowns. So, price and quantity cannot both be denoted by the same symbol, say x.

**Solving:** Using algebraic manipulation (refer previous notes) to express a variable in terms of numbers or other variables or parameters (we will come to parameters later). For example: solving the equation x+5=6 would entail x=6-5=1. But one may also want to take an equation like x+y=6, and solve *for* x. This would mean just expressing x in terms of y and other numbers. So, x=6-y, would be the result.

**Inequalities:** This is a mathematical statement that compares the position of two expressions on the real number line and indicates which one is greater (i.e. further to the right) on the number line. The symbols used are >, <, ≥, and ≤ .

The phrases 'greater than' and 'lesser than' can be confusing because what we mean is 'more positive' or 'less positive'. For example, when comparing a debt of Rs 10,000 with an asset of Rs 500, which is greater? Of course, the debt is larger, but when expressed in terms of the total assets, where the loan comes in as a negative asset, we will write
-10,000 < 500

This implies that the situation with the debt will be further to the left on the number line, i.e. is 'less positive' than the situation with the asset.

Like one solves an equation, one can also solve an inequality. The answer would be again in terms of a range rather than a particular value. So, if x+5<6, then x<6-5, or, x<1.

The rules of manipulation of equations apply the same way for inequalities with one exception. If we multiply by a negative number, then the inequality reverses. It makes sense if you see this using numbers. We know that 2>-3. If we multiply both sides with -1, the LHS becomes -2, and the RHS becomes 3. Hence we get $-2<3$. This is an important thing to keep in mind while working with inequalities. One way to avoid confusion is to only multiply or divide by positive numbers!

**Absolute value:** In many applications, we are not interested in the sign (negative or positive) of a term but in its magnitude, i.e. actually how large or small it is and not whether it is more or less positive. For example, one may want to know how the height of students in this class is different from the average height of students in the University. Here I am not interested in whether the students are taller or shorter, but if they are different. So, if the average height in the University is 160cm, and the height of a student is 170 cm, then the difference is denoted by $|170-160|=10$cm. If the height of the student was 150cm, then the difference would still be $|150-160|=10$cm. Hence, $|x|=x$ if $x\geq 0$, and $|x|=-x$ if $x<0$ (since $-x$ would be positive if $x$ is negative)

**Question:** In the same example, if a student's difference in height from the average is less than 5cm, then what are the possible values for their height? What if their difference is more than 12cm?

$|x-160|<5$

There are two possibilities: if x is more than the average
$x-160>0$, then $x-160<5$, $x<165$

If x is less than the average
$x-160<0$, then $-(x-160)<5$, or, $-x+160<5$, or, $-x<-155$, or $x>155$

Therefore $155<x<165$

$|x-160|>12$

$x>172$, $x<148$

**Equations with more than one variables:**

As we saw earlier we can 'solve' equations with more than one variables by expressing one variable in terms of the others, but we cannot actually find their values in terms of numbers.

$x+y=6$, or, $x=6-y$

But if we know that $y=2$, then we know that $x=6-2=4$.

Hence, an additional piece of information can help us find the values of the variables involved. That additional information need not be as simple as giving the value of the other variable. Instead of that we could have been told that x and y are such that x-y=2. Then we know that x=2+y.

If this is true, and also x=6-y, then we know that y has to be such that 2+y=x=6-y, or, 2y=4, or y=2, and x=4.

When we have multiple equations with the same variables that are true at the same time, we call them **simultaneous equations.** Having number of equations equal to the number of unknowns is a necessary condition to find the value of the unknowns.

**Question:** Two individual go to the market to buy fruits. They buy apples and bananas. The first person spends a total of Rs 50 and buys 4 apples and 15 bananas. The second person spends a total of Rs 100 and buys 8 apples and 30 bananas. Can you find the price of apples and bananas?

Answer: We cannot! Having 2 equations for two unknowns is necessary but not sufficient. In a future chapter we will see how we can find out even before solving the equations whether or not we will be able to find the values of the unknown variables.

**Question:** Now suppose a third individual goes to the market and buys 6 apples and 20 bananas and spends Rs 70. Can we now find the prices of the fruits?