Graphs

We already have a ‘graphical’ representation of real numbers in the form of a line. We need to define a point on this line as 0, also known as the origin, and then define a scale i.e. what length will correspond to 1 unit, and finally define a direction i.e. which direction would be increasing or positive. This gives us the real number line and any real number is a point on this line.

As we had seen earlier, sometimes one number is not sufficient to denote a quantity. For example, location on a map is denoted by two numbers - latitude and longitude. The price level in an economy is shown by the prices of all commodities consumed by an average consumer. The inputs used by a factory may be denoted by multiple numbers: the number of workers, the number of machines, the amount of fuel used etc. These collections of real numbers are called **vectors.** For example consider a factory that has 6 full time workers and consumes 30 litres of fuel. The inputs for this factory can be written in the form of a vector as (6,30). Note that the order in which the numbers are written and their respective units need to be known beforehand if we are to interpret the vector. We can now compare this with another factory that is as (5,45). We can see that the second factory uses less workers and more fuel.

Since each of these two quantities - workers and fuel - are real numbers, we can represent each of them on their own number line. So, there is a number line for workers where the first firm is at 6 and the second at 30. There is a number line for fuel where the first firm is at 5 and the second at 45.

Since any surface consists of two independent dimensions, we can represent each of these lines as one dimension and show the location of a firm in two-dimensional space. Each number line now becomes an **axis.** We typically draw one axis as horizontal and the other as vertical. This is for convenience and according to convention - nothing prevents us from drawing them tilted! So, if we represent the number of workers on the horizontal axis and amount of fuel on the vertical axis. Can you show the points that would donate the two firms, let us call them A and B, on the graph?

Question: Why do the two zeroes have to coincide?
[They don’t have to. It is just easier to interpret if they do]

Since we have a maximum of three dimensions available to us, we can only show a vector that has three quantities in a physical graph. Vectors with more quantities, eg a price vector, cannot be shown graphically. But we can use other aspects of a graph in creative ways to express more quantities. See an interesting example here

Question: What are the two axes?
[Income and life expectancy]

Q: How many quantities are being represented in the animated graph?
[Four: Income, Life expectancy, population size, and time]

Despite this, we will see that we can do some very useful things with the quantities that are on the axes that we cannot do with others.

Let us not think of A and B as two firms but as two possible technologies to produce a given quantity of output. A is a labour intensive technology and B is a capital intensive technology. Now, initially let labour be cheap and fuel be expensive. Let the price of labour be 1000 per worker and the price of fuel be 100 per litre. Which technology would a firm choose?

Cost of A = 6000+3000=9000
Cost of B= 5000+4500=9500

A will be chosen.

Now if the price of fuel decreases to 50 per litre, then?

Cost of A = 6000+1500=7500
Cost of B = 5000+2250=6250

B will be chosen.

You would have seen a similar discussion in the second unit of the Introduction to Economics 1 course as one of the explanations for the Industrial Revolution. Now, let us try to answer a slightly different question. As fuel price starts falling from 100, at what point does technology B become cheaper?

We can answer this question algebraically but let us try to do it using the graph.

First, we want to see at the initial price, what are all the possible technologies that would have cost the same as A. The curve joining all such technologies is called an **isocost curve**. As we have said, a technology is a vector where the first element denotes number of full time workers and second denotes the amount of fuel. So if we write this as a vector of variables (x,y), then x will denote workers and y fuel.

At the initial price of 1000 per worker and 100 per litre of fuel, the cost of technology (x,y) would be 1000x + 100y. We want to get all the possible values of x and y that will give the total cost 9000. Hence we get an equation

1000x + 100y = 9000

Obviously x=6 and y=30 i.e the point denoting the vector (6,30) will satisfy the equation but we want to know all the other possible values. Eg,(9,0). Or, (0,90) Or, (4.5, 45) (what does it mean

to have 4.5 workers?). There are infinitely many combinations possible. So, while we cannot write them down, we can show them on a graph.

When we plot these points on a graph it becomes clear that these are all on a straight line. Equations whose graphs are straight lines are called **linear equations.** [In case you were wondering a straight line is also a curve - it is one of many possible curves. That is where the mathematical meaning of these terms differs from the English meaning].

Question: What makes an equation linear? Are all equations linear?

As an example, if we want to plot a curve for xy=4, it will go through the points (1,4), (4,1) and (2,2). This is clearly not a straight line.

Question: How do we draw the graph of any linear equation? Do we alway have to guess points?

The important thing to know about a straight line is that you need two points to draw it. So, a very easy way to draw it is by using **intercepts.** These are points where the line meets the two axes. In our case, the intercepts also mean something economically. The isocost curve intersects the vertical axis at (0,90), and the horizontal axis at (9,0). These indicate the points that correspond to technologies where only fuel and only workers are used. This means that if a firm had to spend the same amount of money as technology A and use only workers, they would use 9 workers.

Now, instead of thinking about an equation, if we think about the inequality

$1000x + 100y > 9000$

Q. How would this be shown on the graph?
[It will be the region above the first isocost curve]

You can see that point B lies in this region and hence will cost more and will not be chosen.

If we repeat the same exercise with the price of the fuel being 50, we will find the opposite result [This can be done in class if time is available or you can show it on the board.]

Now let us think about the question asked earlier. At what price of fuel does the choice of technology switch from A to B. We can see that the isocost for the second case is **steeper**. So it is something about the **slope** of the graph that is deciding which one is preferred.

The slope of a graph can be easily obtained by writing it in the following form

$y=mx+c$

Here, x and y are variables, but m and c are parameters. The slope is denoted by m and c is the vertical or y-intercept.

If we change equation 1 to this we get

y= - 1000/100 x + 9000/100=-10x + 90

We already knew that the vertical intercept is 90, but now we also know that the slope is -10.
Q. What is the economic interpretation of the slope?
[It is the relative price between workers and fuel. It is ten times more costly to hire one worker than buy one litre of fuel]

Q. What does the negative sign indicate?
[At the same cost if you but more of fuel you can have fewer workers]

Note that if we make an isocost with the same prices but at a different cost, say through point B with a cost of 9500, the slope remains the same but the intercept changes.

Now, at the new prices, the slope can be calculated even without writing the equation of the isocost. Since it is relative price, the slope has to be -1000/50=-20

We can see graphically that the choice of technology will switch when the slope of the isocost will be equal to the slope of the line joining points A and B.

Q So, how do we find the slope of the line joining points A and B?
[m=y1-y2/x1-x2=-15]

Therefore the slope has to be -15, and hence the price of fuel has to be approx 66.6 for the choice to flip.

Class Exercise: We will see how to draw graphs of straight lines on spreadsheets and how to use them to solve equations.

Let us take a simple equation: 5x+2y=12

Now, to draw it using a spreadsheet it is very useful to get it into the slope-intercept form

So, we get y=-5/2x + 6 ……. (2)
Now, label the first column x and the second column **y.**

In the first column put some values of x say 1,2,3,4,5. In the second column insert a formula that calculated y according to equation (2).

Once you have done that. Select both columns and insert an x-y scatter plot. Add trendline and equation to get the final graph.

In the same graph you can add other lines as well. Let us draw the line for 3x-2y=12

This becomes y=3/2x-6

You can add a third column. Call it y2. Keep x the same. Insert the formula and add the graph.