Functions

Q. What are some functions that you have seen in economics already?

[Production function, Utility function]

And there are many more - demand function (we will see that the demand curve is drawn 'wrong'!], profit function etc.

Q. So, what is a function?

There are two important terms: an independent variable or variables - sometime called the argument. For a production function this would be the amount of labour used, and if they are other inputs they would correspond to more independent variables. And **one** dependent variable - for a production function, this would be the output. The function should tell us what the value of the dependent variable would be for all possible values of the independent variables (we will come back to what we mean by possible. For example, a negative value of labour is not possible and hence a production function need not tell us what the output will be for a negative value of labour.)

A function is written in the following way: $y=f(x)$

Here, y is the dependent variable, x is the independent variable and f(.) is the function. For example there could be two weavers having two different production functions that define output (y) as a function of time used (x).

$y=f(x)=2x$
$y=g(x) = 3x$

Therefore, $f(1)=2$, $f(2)=4$, $g(3)=9$ and so on.

The dependent and the independent variables are the same but the different relationships between them are depicted by the two different functions f(.) and g(.) for these two weavers.

Q. If a person walks with a **speed** of 10km/hr, can you write down distance covered as a function of time.

If the function is defined over only one independent variable, we can easily represent it on a graph with the independent variable on the horizontal axis and the dependent variable on the vertical axis. This is by convention but it has some important implications on interpreting the graph. So, for a production function, we would put labour, or time used on the horizontal axis and output on the vertical axis.

Q. Notably, the demand curve goes against this convention - can you say why?

So, what is not a function?
If I say profit increases with price, that is not a function - because it does not tell me what the profit will be for a given price. If I say profit is equal to revenue minus cost, then it is a function: profit is a function of revenue and cost as for any given values of revenue and cost I can find the profit.

Q. In Bangalore the average speed of cars on the road is anywhere between 10 and 30 km/hr. Can we use this to write distance as a function of time?

[Ans. No. The dependent variable can have multiple values for a given value of the independent variable here so it is not a function.]

The independent variable can be of different kinds, and that will decide what values of the independent variable the function is defined for. For example, if the independent variable is the number of students in the classroom (n) and the dependent variable is the time taken to grade assignments (t). We can say

$t=f(n)$ where n is a natural number (or sometimes this is written as $n \in \mathbf{N}$.

For a production function we may say
$y=g(x)$, where $x \geq 0$

The values of the independent variable for which the function is defined is called the **domain** of the function. In most cases, this is not explicitly mentioned. We then assume the most reasonable interpretations given the context.

The dependent variable may not necessarily be defined over the same set of values as the independent variable. For instance, in the example of number of students and time to grade, while the independent variable is always a natural number, the dependent variable could be any positive real number. This is called the **range** of a function.

Q. There are two jobs - one with salary Rs 30000 and another with Rs 20000. The first one is only given to people with age 30 and above, while the second one is only for people below age 30. Define salary as a function of age, draw its graph and write down what the domain and range are.

[Key point is that range consists of only two values 20000 and 30000]

Some useful terms

Increasing / decreasing function: There are some functions that are always increasing. For example: the total number of covid cases in Bangalore as a function of time. Without even writing down an actual algebraic function, we know that it will always be increasing. In most (not all) cases, production functions are also increasing.

Q. The **slope** of a function can tell us whether it is increasing or decreasing. How?

[Positive slope is increasing. Negative slope is decreasing.]

Inverse of a function: In one of the questions above, we wrote distance (y) as a function of time (x) when the speed was given. y=f(x)=10x. We can also write time taken to travel a given distance. x=y/10. We call this as the inverse of the first function or $x=f^{-1}(y)$

This is important in economics. Sometimes from a demand function q=g(p), we want to know the inverse demand function i.e. what should be the price to generate a given level of demand. $p=g^{-1}(q)$.

But not all functions are invertible. Take for example a function that gives the price of fruits in a market as a function of time in the day. It is low in the morning and then it rises through the day as more people come to buy and then it falls in the end because the seller wants to get rid of the remaining fruits. Hence the graph of price versus time will look like and inverted U. Can we now write time as a function of price i.e. for a given price the function should tell us what time it was? It doesn't work because the same price could have happened in the morning or the evening. Hence, we can't obtain a unique time given a price and therefore the price function is not invertible.

Some commonly used functions:

Linear functions

These are called linear as their graphs are straight lines. They are of the form

$y= f(x)=mx+c$

As we had discussed earlier, the slope is given by m. Hence, the function is increasing or decreasing based on the sign of m. A linear function is always invertible as every value of y is generated by a unique value of x.

$x= f^{-1}(y)=(y-c)/m$

Note that the slope of the inverse function is 1/m.

Piecewise linear functions
Recall the production functions for food and labour we had encountered in the first assignment. Let us take the production function for food from labour. There was a fixed cost of 7 and we wrote the function as

f=g(l)=2(l-7)

So, if l=7, f=0,; l=8, f=2; l=9, f=4, and so on. But what if l is less than 7? What should be the production if we cannot meet the food cost? It should be 0. Hence, if we want to define the production function for the complete range of l, which is all values of l>=0, we should write

f=0 if l<7
=2(l-7) if l>=7

[Draw the graph]

This is called a piecewise linear function as it is made up of different pieces where each piece is a linear function. Is the function invertible? (No, because if f=0, l could take any value less than 7)

Q. Write and draw the piecewise linear function for the production function of labour using food from the same question.

Quadratic functions

As we saw with quadratic equations, a quadratic function is one in which the highest power of the independent variable is 2. The simplest quadratic function is
$y=f(x)=x^2$

Q. Use a spreadsheet to draw a graph of this function

Once done, also draw graphs of $g(x)=x^2+2x+1$ and $h(x)=3x-x^2$

Q. Are they increasing/decreasing? Invertible/non-invertible?