Review of function of multiple variables

- independent variable is a vector of many variables
- dependent variable is only one variable
- With two independent variables - can depict in 3 D graph
- Can slice the graph by fixing various values of one variable
- Level curves

Consider a production function q=f(x,y)=4xy. Where x is labour and y is capital.

Q. What is the marginal product of labour when capital is (a) 0, (b) 1, (c) 2?
[0,4,8]

The important thing to note is that the marginal product of labour depends on the value of capital.

This idea of finding the slope of a function with respect to one variable, while keeping the values of the other variables fixed, is called partially differentiating the function, and the slope is called the **partial derivative**.

- Derivative of a single variable function

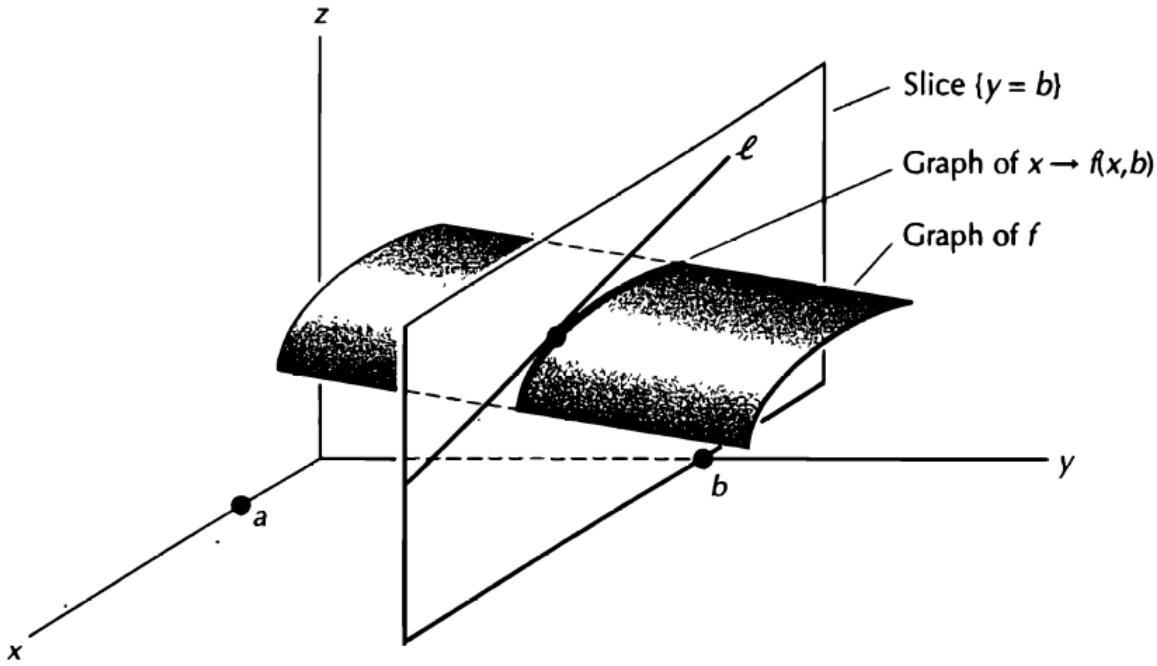
$$\frac{df}{dx}(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

- Partial derivative of a function of multiple variables

$$\frac{\partial f}{\partial x_i}(x_1^0, \ldots, x_n^0) = \lim_{h \to 0} \frac{f(x_1^0, \ldots, x_i^0 + h, \ldots, x_n^0) - f(x_1^0, \ldots, x_i^0, \ldots, x_n^0)}{h}$$

Note the curved d's in the notation.

Below is a graphical representation of the partial derivative;



The graph of $x \longmapsto f(x, b)$ on the slice $\{y = b\}$.

Finding the partial derivative

The procedure for finding the partial derivative is to treat the function as a function with a single variable, and treat all other variables as constants.

So the partial derivative of 4xy is just 4y, as both 4 and y are constants.

Now try to find the partial derivatives as given below

$$\frac{\partial}{\partial x}(3x^2y^2)$$

$$\frac{\partial}{\partial x}(7y)$$

$$\frac{\partial}{\partial x}(3x^2y^2 + 4xy^3 + 7y)$$

$$\frac{\partial}{\partial y}(3x^2y^2)$$

$$\frac{\partial}{\partial y}(3x^2y^2 + 4xy^3 + 7y)$$

Answers:

$$\frac{\partial}{\partial x}(3x^2y^2) = 2x \cdot 3y^2 = 6xy^2.$$

$$\frac{\partial}{\partial x}(7y) = 0.$$

$$\frac{\partial}{\partial x}(3x^2y^2 + 4xy^3 + 7y) = 6xy^2 + 4y^3.$$

$$\frac{\partial}{\partial y}(3x^2y^2) = (2y)(3x^2) = 6x^2y.$$

$$\frac{\partial}{\partial y}(3x^2y^2 + 4xy^3 + 7y) = 6x^2y + 12xy^2 + 7.$$

Exercise:

- Compute both partial derivatives of
  - x^2+y^2
  - $4x^2y - 3xy^3 + 6x$
  - e^{2x+3y}

- Let the utility function of an individual over milk (x) and tea (y) be u(x,y) = √(xy). What is the marginal utility of milk when the individual is already consuming 4 units of milk and 1 unit of tea?

- Wage as a function of education (x) and experience(y) is given as w(x,y)=100x + 20y + 10xy. For a person with 10 years of education, how much does their wage increase with every additional year of experience?